

**Spin and its evolution for isolated neutron stars and X-ray binaries:
the determination of the 'Diffusion coefficients' and SPINDOWN
THEOREM.**

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Abstract

In this work we give detail consideration of the possible scenario of evolution of isolated neutron stars (INSs) and determine some characteristics of X-ray pulsars from their spin period evolution.

The new points of our consideration are:

–we give additional arguments for the short time scale of the Ejector stage ($\approx 10^7 - 10^8$ yrs).

–we proposed specific SPINDOWN THEOREM and give some arguments for its validity. Discovery of accreting INSs will means that the SPINDOWN THEOREM is true.

–we consider firstly evolution of spin period of a NS on the Accretor stage and predict that its period $\geq 5 \cdot 10^2$ sec and INSs can be observed as pulsating X-ray sources.

–we modeled accretion onto an INS from the interstellar medium in the case of spherical symmetry for different values of the magnetic field strength, ambient gas density and NS's mass. The periodic sources with P from several minutes to several months can appear.

–we consider new idea of stochastic acceleration of very old NSs due to accretion of turbulized ISM.

–last point means that INSs can be spin up and spin down with equal probability.

–using the observed period changes for four systems: Vela X-1, GX 301-2, Her X-1 and Cen X-3 we determined D , the 'diffusion coefficient', – parameter from the Fokker–Planck equation.

–using strong dependence of D on the velocity for Vela X-1 and GX 301-2, systems accreting from a stellar wind, we determined the stellar wind velocity. For different assumptions for a turbulent velocity we obtained $V = (660 - 1440) \text{ km} \cdot \text{s}^{-1}$.

–we also determined the specific characteristic time scales for the 'diffusion processes' in X-ray pulsars. It is of order of 200 sec for wind-fed pulsars and 1000-10000 sec for the disk accreting systems.

1 Introduction

Isolated neutron stars (INSSs) which are not observable as radiopulsars became recently to attract much interest. Treves & Colpi (1991) suggested that INSSs accreting from interstellar medium (ISM) can be observed in UV and X-rays by *ROSAT* and several of them were observed.

For main-sequence stars there are two the most important parameters which are needed for description of their evolution: the mass and the age. For NSs there are also two important parameters: the period and the gravimagnetic parameter, y .

The gravimagnetic parameter was firstly introduced by Davis & Pringle (1981) as:

$$y = \frac{\dot{M}}{\mu^2} \quad (1)$$

There are four possible states of a NS in low density plasma: E (Ejector), P (Propeller), A (Accretor) and G (georotator), depending on relations between four characteristic radii: R_l , radius of the light cylinder, R_{st} , stop radius, $R_G = (\frac{2GM}{v_\infty^2})$ -radius of gravitational capture and $R_c = (\frac{GM}{\omega^2})^{1/3}$ -radius of corotation.

As the result we have two critical periods: P_E and P_A , separating different stages of NS. If $P < P_E$ we have Ejector, if $P_E < P < P_A$ we have the Propeller stage and if $P > P_A$ and $R_{st} < R_G$ -the Accretor stage. In some cases it may be $P > P_A$ but $R_{st} > R_G$ and accretion is not possible because Geo-like magnitosphere is formed.

Usially a track of a NS can be represented as a monotonic spin-down, consequntly passing through three stages:

$$\begin{aligned} E &\longrightarrow P \longrightarrow A \\ E &\longrightarrow P \longrightarrow G \end{aligned}$$

The stage of Georotator can take place if:

$$y < y_G = \frac{v_\infty^7}{(2GM)^4} \quad (2)$$

On the Fig.1 we present examples of the NS evolutional tracks.

In this poster we show some results, concerning periods of NSs, their evolution, determination of the stellar wind velocities in X-ray pulsars and INSSs. Mostly we are based on three our articles (Popov 1994, Lipunov and Popov 1995a,b)

2 The Spindown theorem.

The equation of evolution of INSSs can be represented in the form of the spin-down equation (Lipunov 1982):

$$\frac{dI\omega}{dt} = -\frac{k_t\mu^2}{R_t^3} \quad (3)$$

where k_t —dimensionless factor, R_t —the scale of the interaction between the magnetic field of the NS and surrounding plasma.

For the next consideration we proposed some kind of astrophysical theorem from the general point of view.

THE SPINDOWN THEOREM.

The duration of the Propeller stage under all other constant conditions is smaller than duration of the Ejector stage:

$$t_P \leq t_E, \quad (SDT)$$

Arguments.

I). For the most investigated propeller regimes the spin-down coefficient, k_t , slowly depends on the spin frequency, ω , and as result the duration time of the Propeller stage is determined by the initial (not final, as for Ejector) period of the NS!

II). The interaction between the magnetic field of the NS and accreting plasma in each case is more effective than the interaction between the magnetic field and vacuum (magneto-dipole spin-down). Realy, we have the relation:

$$\beta = \frac{\left(\frac{dI\omega}{dt}\right)_E}{\left(\frac{dI\omega}{dt}\right)_P} = \frac{2}{3k_t} \cdot \left(\frac{R_A}{R_l}\right)^3. \quad (4)$$

Because $(R_A/R_l)^3$ is a very small parameter we can expect that $\beta < 1$, for a very large range of k_t .

Conclusion.

If 1).and 2). are true then the duration of the Propeller stage is less or equal than the duration of the Ejector stage.

If we believe in the SPINDOWN THEOREM we conclude that after 10^{7-8} yrs the INS in real interstellar medium comes to the Accretor regime.

We have *strong observational argument* for the SPINDOWN THEOREM: the X-ray pulsar X Per is a very slowly rotating NS (period $p=835$ s) associated with young massive companion (age of companion $\approx 10^7$ yrs) and has a very low accretion rate (about $\approx 10^{13-14}$ g/s approaching to interstellar medium). *We can not understand the existance of this X-ray pulsar if the propeller mechanism is not effective!*

3 Stochastic acceleration of INSSs.

Now we will discuss the situation with very old INSSs and their periods. Let us consider the fluctuations of the period of the NS. Suppose that the change in the angular velocity of the accreting star occurs under the action of a random torque:

$$\frac{d\omega}{dt} = -\frac{k_t \mu^2}{IR_c^3} + \Phi \quad (5)$$

here Φ is the fluctuating torque due to accretion from turbulized ISM. We can introduce the scalar potential V (see Lipunov 1992):

$$\frac{k_t \mu^2}{IR_c^3} = \nabla_\omega V(\omega) \quad (6)$$

The distribution function among frequency, $f(\omega)$, can be received as a result of stationary solution corresponding the Fokker–Planck equation:

$$f(\omega) = N \cdot \exp(-V(\omega)/D) \quad (7)$$

here D —the diffusion coefficient. For our case we have:

$$V(\omega) = \frac{k_t \mu^2}{3GM} |\omega|^3 \quad (8)$$

and

$$D \approx \frac{1}{2} \left(\frac{\dot{M} v_t R_t}{I} \right)^2 \frac{R_G}{v} \quad (9)$$

where v_t and R_t —velocity and length of the turbulence. We must put $R_t \approx R_G$, and after some calculations we obtain:

$$D \approx 7.6 \cdot 10^{-18} \rho_{-24}^2 v_6^{-13} v_{t_6}^2 I_{45}^{-2} \left(\frac{M}{M_\odot} \right)^7 \quad (10)$$

Assuming $V(\omega) \approx D$ we obtain the estimate of the possible period of INS due to stochastic acceleration:

$$\begin{aligned} P &= 2\pi \left(\frac{k_t \mu^2}{3GMID} \right)^{1/3} \approx \\ &\approx 5 \cdot 10^2 k_t^{1/3} \mu_{30}^{2/3} I_{45}^{1/3} \rho_{-24}^{-2/3} v_6^{13/3} v_{t_6}^{-2/3} \left(\frac{M}{M_\odot} \right)^{-8/3} s. \end{aligned} \quad (11)$$

In principle this estimates show that *real period of INS can be about several hours or days*. But due to the very strong dependence on the velocity, v , and

the turbulent velocity, v_t , at the scale about R_G we cannot give more precise results. We only illustrate that *the old INSs can be accelerated by stochastic angular momentum from the turbulent ISM. In this case we can see with equal probability spin-up and spin-down INSs!*

Spin-up and spin-down time can be estimated as:

$$\begin{aligned} t_{su} \approx t_{sd} &\approx \frac{I\omega}{\dot{M}v_t R_G} = \frac{Iv_\infty^5}{(2GM)^3 \rho_\infty v_t p} \approx \\ &\approx 20 \text{ yrs } I_{45} v_6^5 \rho_{-24}^{-1} \left(\frac{p}{10^5 \text{ s}} \right)^{-1} \left(\frac{v_t}{10^6 \text{ cm/s}} \right)^{-1} \end{aligned} \quad (12)$$

Really v_t is a small fraction of the sound velocity, because in opposite case the energy of turbulence will dissipate in the form of shock waves. From observations it is well known that turbulence has a power spectrum:

$$v_t \sim l^\alpha, \quad (13)$$

(here l is the scale of the turbulent torque with turbulent velocity v_t), but exact value of α is unknown. Very often the turbulence spectra is represented in the form of the Kolmogorov spectrum:

$$v_t^3 = \epsilon l, \quad (14)$$

where ϵ is a constant which characterises energy transfer. If we want to calculate v_t on the scale of R_G we must know v_t on the characteristic scale. Turbulence is observed on scales $10^8 < l < 3 \cdot 10^{20} \text{ cm}$ and for $l = (50 - 150) \text{ pc}$ v_t is of order of 10^6 cm/s (Ruzmaikin et al 1988). we also use another value: $v_t = 10^5 \left(\frac{l}{10^{18} \text{ cm}} \right)^{1/3}$ (Kaplan & Pikelner 1979). so for $R_G = 4 \cdot 10^{14} \text{ cm}$ we have: $v_t \approx 10^4 - 10^5 \text{ cm/s}$. In this case t_{su} and t_{sd} are of order of 200-2000 yrs. In dense clouds ($\rho_{-24} \approx 100$) $v_t \approx 3 \cdot 10^3 - 10^4 \text{ cm/s}$ (Canuto & Battaglia 1985) and $t_{su} \approx t_{sd} \approx 20 - 60 \text{ yrs}$ and NS can be observed in principle.

3.1 Accretion onto OINSs and periodic sources

We modeled accretion onto an INS from the interstellar medium in the case of spherical symmetry for different values of the magnetic field strength, ambient gas density and NS's mass. We tried to verify the idea that if the radius of corotation, R_{co} , is less than the Alfvén radius, R_A , the shell will form around the INS and R_A will decrease to R_{co} and the periodic X-ray source will appear (see Colpi et al., 1993).

Dependence of R_A from t in our model coincides well with the analytic formula from Colpi et al.(1993). *The periodic sources with P from several minutes to several months can appear* (see Popov 1994).

4 The 'Diffusion coefficients' and the stellar wind velocities for X-ray binaries.

Here we shall try to estimate the diffusion coefficients for selected systems (see section "Stochastic acceleration of INSSs") and stellar wind velocities (see details in Lipunov & Popov, 1995a,b). At first we shall estimate D . All variables, except Δt , are known (in principle). Their values taken from Lipunov (1992) are shown in table 1.

Characteristic time Δt for the wind-accreting systems can be determined from the equation:

$$\Delta t \approx 1.7 \cdot 10^4 \alpha^{-2} 10^{2(A+8.5)} L_{37}^{-\frac{12}{7}} \mu_{30}^{-\frac{4}{7}} \text{sec}, \quad (15)$$

where α – a fraction of the specific angular momentum of the Kepler orbit at the magnitospheric radius, A –noise level (see table 2) (de Kool & Anzer 1993).

So we can write equation for D in the form:

$$D = 5.55 \cdot 10^{-19} \mu_{30}^2 \omega^3 \gamma_6^{-1} I_{45}^{-1} \left(\frac{M}{1.5M_{\odot}} \right)^{-1} \text{s}^{-3}. \quad (16)$$

Values of D for four systems are shown in table 3.

For characteristic time in the frequency space we can write:

$$t_{char} = \omega_{char}/\dot{\omega} = \frac{Dp^4}{4\pi^2 \dot{p}^2} \quad (17)$$

We can give a physical interpretation for t_{char} for wind-fed pulsars as a characteristic time of the momentum transfer:

$$\frac{R_G}{v_{sw}} = 400 \left(\frac{M}{1.5M_{\odot}} \right) \left(\frac{v_{sw}}{10^8 \text{cm/s}} \right)^{-3} \text{sec} \quad (18)$$

These characteristic time scales are also shown in table 3.

Now for D we can write:

$$D = 4.38 \cdot 10^{-23} \dot{M}_{16}^2 \eta^2 T_4 \mu^{-1} v_8^{-7} I_{45}^{-2} \left(\frac{M}{1.5M_{\odot}} \right) \text{s}^{-3}. \quad (19)$$

As we see *there is a strong dependence of D on v* . So we can evaluate v (in this case it is the stellar wind velocity, v_{sw}):

$$v_{sw} = 1700 \cdot D_{-24}^{-1/7} \dot{M}_{16}^{2/7} \eta^{2/7} T_4^{1/7} \mu^{-1/7} I_{45}^{-2/7} \left(\frac{M}{1.5M_{\odot}} \right)^{2/7} \text{km/s}. \quad (20)$$

Values of v_{sw} are shown in table 3.

Table 1:

	p, sec	p_{orb} , sec	L, erg/sec	$\mu/10^{30} Gs \cdot cm^3$	$t_{su\,observ}$, yrs	$t_{su\,min}$, yrs
Vela X-1	283	$7.7 \cdot 10^5$	$1.5 \cdot 10^{36}$	3	3000	3000
GX 301-2	696	$3.6 \cdot 10^6$	10^{37}	120	> 100	100
Her X-1	1.24	$1.5 \cdot 10^5$	10^{37}	0.6	$3 \cdot 10^5$	8000
Cen X-3	4.84	$1.8 \cdot 10^5$	$5 \cdot 10^{37}$	5.7	3400	600

Table 2:

	A	L_{max}
Vela X-1	-9.1	$10^{36.8}$
GX 301-2	-8.5	10^{37}

Table 3:

	Δt , sec	γ	D , sec^{-3}	v_{sw} , km/s, ($v_t = 0.1 \cdot a_s$)	v_{sw} , km/s, ($v_t = a_s$)	t_{char} , sec
Vela X-1	$1.5 \cdot 10^4$	$6.3 \cdot 10^6$	$8.7 \cdot 10^{-24}$	848	1442	200
GX 301-2	$1.1 \cdot 10^3$	$2.9 \cdot 10^6$	$2 \cdot 10^{-21}$	656	1120	245
Her X-1	$4 \cdot 10^3$	$6.0 \cdot 10^7$	$4 \cdot 10^{-19}$	—	—	960
Cen X-3	$2 \cdot 10^4$	$9.5 \cdot 10^5$	$4.1 \cdot 10^{-17}$	—	—	8500

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More information on the topic can be found at: <http://xray.msu.su~polar/>. Your comments you can send to:polar@xray.sai.msu.su or ps@sai.msu.su.

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